ECON 702 Macroeconomics I

Discussion Handout 10*

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A General Expression of Uncertainty

- In each period $t \geq 0$, a stochastic event $s_t \in S$ is realized.
- $s^t = (s_0, s_1, \ldots, s_t)$ denotes a history.
- $\pi_t(s^t)$ denotes the probability of a history s^t.
- $\pi_t(s^t \mid s^\tau)$ denotes the probability of history s^t conditional on the realization of s^τ with $t > \tau$.
- We can express variables in the economy in terms of the stochastic state s^t.

Example: Markov chains

Markov Property: A stochastic process $\{x_t\}$ is said to have the Markov property if for all $k \geq 1$ and all t,

$$Prob(x_{t+1}|x_t, x_{t1}, ..., x_{tk}) = Prob(x_{t+1}|x_t)$$

A finite state Markov chain with n possible values is defined by an initial state or initial probabilities in combination with a $n \times n$ transition matrix P such that

$$P_{ij} = Prob(x_{t+1} = s_j|x_t = s_i)$$

Thus, note that

$$\sum_{j=1}^{n} P_{ij} = 1$$

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1. Suppose $s_t \in \{0, 1\}$ follows a Markov process, and $s_0 = 0$ (WLOG). Draw a tree to represent s_t in each $t = 0, 1, 2, 3$. Write s^t for each terminal node.

2. Suppose $Prob(s_{t+1} = 0 | s_t = 0) = .5$ and $Prob(s_{t+1} = 0 | s_t = 1) = .75$.
   Find $Prob(s_3 = 0)$ and $Prob(\sum_{t=0}^{3} s_t = 2)$.

3. Is an AR(1) process a Markov process?

Neoclassical Growth With Uncertainty

Consider the typical Neoclassical growth model, but with an uncertain aggregate state in each period, with histories indexed by s^t with probabilities $\pi_t(s_t)$. The discount rate β, the rate of depreciation of capital δ, and the representative household's utility function $u(c)$ remain constant, but other parameters and variables depend on the state s^t. For generality, we write the production technology $F(s^t, K_t(s^t), L_t(s^t)$ which is CRS. Households exogenously supply labor $L_t(s^t)$ and consume the final good $C_t(s^t)$.

1. Discuss some potential economic meanings for the aggregate state.

2. Write the social planner's problem from time 0 in general.

3. Formulate the Lagrangian of the social planner and derive the Euler equation.