ECON 702 Macroeconomics I

19 April 2024

1 Money in Utility

Consider an infinite-horizon model economy in which households derive utility from consumption, c_t, and real cash holdings, $\frac{m_t}{p_t}$, as some goods can only be bought using cash. Households supply labor, l_t, at price w_t in a competitive labor market and get disutility from work. Besides, they rent out capital, k_t, to the firms at price μ_t in a competitive capital market. Each period using labor income, households can consume, c_t, invest in capital (save), k_{t+1}, keep cash, m_{t+1}, and buy nominal bonds, b_{t+1}. Nominal bonds cost 1 nominal unit in period t and pay R_{t+1} nominal units in period $t+1$. Household preferences are given by:

$$U\left(c_t, l_t, \frac{m_t}{p_t}\right) = \log\left(c_t - \psi\frac{l_t^{1+\eta}}{1+\eta}\right) + \xi\log\left(\frac{m_t}{p_t}\right).$$

Firms produce using Cobb-Douglas technology:

$$Y_t = F(k_t, l_t) = k_t^\alpha (A_t l_t)^{1-\alpha}.$$

Assume that capital fully depreciates during production in each period, $\delta = 1$, and households discount their future with the discount factor $\beta \in (0,1)$.

The government has a money supply rule:

$$m_t = m e^{\hat{m}_t}$$
$$\hat{m}_t = \rho_m \hat{m}_{t-1} + \sigma_m \varepsilon_{m,t-1},$$

where m is the mean of the money supply and $\hat{m}_t$ is a log-deviation that follows AR-1 process. The government has to balance its budget:

$$T_t + R_t\frac{b_t}{p_t} = \frac{1}{p_t}(m_{t+1} - m_t) + \frac{b_{t+1}}{p_t}.$$

1. Write the household problem and derive the first-order conditions.

2. Using the first-order condition derive the Euler equations for money holdings and nominal bonds. Derive the non-arbitrage condition between money holdings and nominal bond holdings. Provide an interpretation.

3. If the nominal gross return on bonds, R_{t+1}, equals 1, does the non-arbitrage condition hold? How does the model parameter ξ affect the answer? Provide an interpretation.

4. If $\xi > 0$, under which condition for R_{t+1} agents hold both types of nominal assets in the equilibrium?

5. Suppose that the decision rule for capital and consumption is $k_{t+1} = \alpha\beta y_t$ and $c_t = (1 - \alpha\beta)y_t$ (Why is it true?). Using the Euler equation for the cash holdings, derive the optimality condition for the price, p_t, as a function of y_t, m_t, and the model parameters.