ECON 702 Macroeconomics I

Discussion Handout 12*

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Consider a small open economy that lasts for two periods, $t = 1, 2$. The representative household has preferences over consumption given by:

$$U(C_1, C_2) = u(C_1) + \beta u(C_2)$$

where $0 < \beta < 1$ is the discount factor. The household is endowed with income Q_1 in period 1 and Q_2 in period 2. The household can borrow and lend in international financial markets at a constant interest rate $r > 0$, and enters period 1 with a net foreign asset position of B_0^*.

1. Derive the household's intertemporal budget constraint.
2. Write down the household's optimization problem, and solve for the Euler equation.
3. Suppose that $u(c) = \sqrt{c}$, $\beta = \frac{1}{1+r} = 1$, and $Q_1 = 9$, $Q_2 = 16$, $B_0^* = 0$. Find the fraction γ of lifetime income that the country would be willing to give up to participate in international markets.
4. How does your answer to the previous question change with $u(c) = -\frac{1}{c}$? Give economic intuition.
5. Find the trade balance and current account in the setting of part 3.
6. What about when $r = \frac{1}{15}$, and $\beta = \frac{1}{1+r} = \frac{15}{16}$? What about when $B_0^* = -5$? Verify in this case that $CA_t = \Delta NFA_t$.

Solutions:

1. The period 1 budget constraint is $C_1 + B_1^* = Q_1 + (1+r)B_0^*$ and the period 2 budget constraint is $C_2 = Q_2 + B_1^*(1+r)$. Combining these we get the intertemporal budget constraint:
$$C_1 + \frac{C_2}{1+r} = (1+r)B_0^* + Q_1 + \frac{Q_2}{1+r}$$

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2. The household's optimization problem is:

$$\max_{C_1,C_2} \quad u(C_1) + \beta u(C_2) \text{ s.t.} \qquad C_1 + \frac{C_2}{1+r} = (1+r)B_0^* + Q_1 + \frac{Q_2}{1+r}$$

The first order conditions give the familiar Euler equation:

$$u'(C_1) = \beta(1+r)u'(C_2)$$

3. If the country is in autarky, they just consume their GDP and get utility

$$\sqrt{Q_1} + \sqrt{Q_2} = \sqrt{9} + \sqrt{16} = 7$$

Euler equation tells us that $C_1 = C_2$ when the country participates in international markets. This gives us

$$C_1 = C_2 = \frac{Q_1 + Q_2}{2} = \frac{9+16}{2} = 12.5$$

This gives the household utility of

$$\sqrt{12.5} + \sqrt{12.5} = 2 * 5/\sqrt{2} = 5\sqrt{2} > 7$$

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If the household gives up a fraction γ of their lifetime income $(Q_1 + Q_2)$, their lifetime income is $(1-\gamma)(Q_1+Q_2) = 12.5(1-\gamma)$. Then,

$$C_1 = C_2 = 12.5(1-\gamma)$$

and they get utility

$$\sqrt{(1-\gamma)12.5} + \sqrt{(1-\gamma)12.5} = 2\sqrt{12.5 * (1-\gamma)}$$

For the household to be indifferent,

$$2\sqrt{12.5 * (1-\gamma)} = 7$$

$$\implies 12.5 * (1-\gamma) = 49/4$$

$$\implies \gamma = \frac{1}{50} = 2\%$$

4. In autarky, the household gets utility

$$-\frac{1}{9} - \frac{1}{16} = -\frac{25}{144}$$

In world markets, household has

$$C_1 = C_2 = 12.5 * (1-\gamma)$$

so they get utility

$$-\frac{1}{12.5(1-\gamma)} + -\frac{1}{12.5(1-\gamma)} = -\frac{4}{25(1-\gamma)}$$

If they are indifferent,

$$-\frac{4}{25(1-\gamma)} = -\frac{25}{144}$$

$$\implies \gamma = 1 - \frac{144*4}{25*25} = .0784 = 7.84\%$$

The household is willing to give up a much larger fraction of their lifetime income because they are more risk averse.

5. We have $CA_t = r * B^*_{t-1} + (Q_t - C_t)$.

   Initially, since $r = 0$, we have $CA_t = TB_t$, so $CA_1 = 9 - 12.5 = -3.5$, $CA_2 = Q_2 - C_2 = 3.5$.

6. Next, when $r = 1/15$, and $B^*_0 = 0$, $C_1 = C_2 = (Q_1 + Q_2/(1+r))/(1 + \frac{1}{1+r} = (9+15)/\frac{31}{16} = 12.39$

$$CA_1 = B^*_1 = Q_1 - C_1 = 9 - 12.39 = -3.39$$

$$CA_2 = r * B^*_1 + Q_2 - C_2 = -3.39/15 + 16 - 12.39 = 3.39$$

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Finally, when $B^*_0 = -5$, we have

$$C_1 = C_2 = \frac{1}{2}((1+r)B_0 + Q_1 + \frac{Q_2}{1+r} = (-5+9+16)/2 = 10$$

$$CA_1 = TB_1 = Q_1 - C_1 = 9 - 10 = -1$$

$$CA_2 = TB_2 = Q_2 - C_2 = 16 - 10 = 6$$

Note that

$$\begin{aligned} &CA_t = (Q_t - C_t) + rB^*_{t-1} \\ \implies &CA_t = B_t - B_{t-1} \qquad \text{from budget constraint} \end{aligned}$$

So we have $B^*_1 = Q_1 + B^*_0 - C_1 = 9 - 5 - 10 = -6$, $B^*_2 = 0$

$$CA_1 = -1 = -6 - (-5) = B^*_1 - B^*_0$$

$$CA_2 = 6 = -(-B^*_1)$$