ECON 702 Macroeconomics I

Discussion Handout 2*

Solutions

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Content review

- **Gross Domestic Product** (GDP) is the market value of all final goods and services produced in a country in a given period of time (usually in a year). It is a statistic which is meant to represent the size of an economy.
- The 3 equivalent methods of calculating nominal GDP (Y) are
  - **Production approach:** Y = sum of value added across industries
  - **Expenditure approach:** $Y = C + I + G + (X - M)$
  - **Income approach:** Y = employee compensation + net operating surplus + tax revenue less subsidies + depreciation
- We use different price indices to calculate inflation. Let p_{it} and q_{it} be the price and quantity of good i at time t.
  - Laspeyres price index (CPI):
  $$L_0^t = \frac{\sum_i p_{it} q_{i0}}{\sum_i p_{i0} q_{i0}}$$
  - Paasche price index (GDP deflator):
  $$Pa_0^t = \frac{\sum_i p_{it} q_{it}}{\sum_i p_{i0} q_{it}}$$
  - Fisher ideal index:
  $$F_{t-1}^t = (L_{t-1}^t * Pa_{t-1}^t)^{\frac{1}{2}}$$

*Teaching Assistants: Anna Lukianova (Email:lukianova@wisc.edu) and John Ryan (Email: john.p.ryan@wisc.edu). Based on the lecture notes by Jesus Fernandez-Villaverde and Dirk Krueger.

- The household problem is to maximize lifetime utility subject to period budget constraints:

$$\begin{aligned} \max_{\{c_t,,a_{t+1}\}} \quad & \sum_{t=0}^{T} \beta^t u(c_t) \\ \text{s.t.} \quad & c_t + a_{t+1} = w_0 + (1+r_t)a_t \\ & a_0 \text{ given }, \\ & a_T = 0 \end{aligned}$$

  We can use the Lagrangian, $\mathcal{L} = \sum_{t=0}^{T} \beta^t u(c_t) + \lambda_t(w_t + (1+r_t)a_t - c_t - a_{t+1}$, and take first order conditions to solve the household problem.

Table 1

(a) Steel Inc.

Domestic Revenue	\$100
Export Revenue	\$50
Wages	\$70
Taxes	\$40

(b) Standard Gas

Revenue	\$120
Wages	\$60
Loan Interest	\$10
Taxes	\$30

(c) Motor Company

Revenue	\$250
Wages	\$55
Cost of Steel	\$100
Machinery investment	\$75
Taxes	\$20

(d) Government

Tax Revenue	\$105
Wages for Building Roads	\$35
Military Wages	\$70

(e) Household

Wage Income	\$290
Profit Received	\$60
Interest Income	\$10
Capital Income	\$75
Taxes	\$15
Import Expenditures	\$75

Exercises

1. Consider the following economy which produces steel, gasoline, cars, has 2 households and a government. The following tables represent the economic transactions of these agents in the economy. Calculate GDP using the production, expenditures, and income approach.

   Solution:

   - Production approach: $Y = (\$100+\$50)+(\$120)+(\$250-\$100)+(\$35+\$70) = \$525$ VA (Steel) + (Gas) + (Cars) + (Government)

   - Expenditure approach: $Y = C + I + G + (X - M) = (\$120 + \$250) + (\$75) + (\$35 + \$70) + (\$50 - \$75) = \$525$
   - Income approach: $\$290 + \$60 + \$10 + \$75 + (\$40 + \$30 + \$20) = \$525$

2. Now consider an economy with milk, bread and fruit in the year 2022 and 2023. Use the following table of quantities and prices to calculate nominal GDP in each year and the 2023 real GDP with 2022 as the base year. Calculate the Laspeyres, Paasche and Fisher ideal price indices, and use these to compare their estimates of inflation.

	2022	2023
Quantity of Milk	10	12
Price of Milk	\$4	\$5
Quantity of Bread	25	26
Price of Bread	\$2	\$2.50
Quantity of Fruit	15	12
Price of Fruit	\$3	\$6

Solution:

- Nominal GDP 2022: $\$4 * 10 + \$2 * 25 + \$3 * 15 = \$135$
- Nominal GDP 2023: $\$5 * 12 + \$2.50 * 26 + \$6 * 12 = \$197$
- Real GDP 2022: \$135
- Real GDP 2023: $\$4 * 12 + \$2 * 26 + \$3 * 12 = \$136$
- Laspeyres price index:

$$L_0^t = \frac{\sum_i p_{it} q_{i0}}{\sum_i p_{i0} q_{i0}} = \frac{\$5 * 10 + \$2.50 * 25 + \$6 * 15}{\$135} = 1.5$$

- Paasche price index:

$$Pa_0^t = \frac{\sum_i p_{it} q_{it}}{\sum_i p_{i0} q_{it}} = \frac{\$197}{\$136} \approx 1.448$$

- Frische ideal index:

$$F_{t-1}^t = (L_{t-1}^t * Pa_{t-1}^t)^{\frac{1}{2}} = \sqrt{1.5 * 1.448} \approx= 1.474$$

The Laspeyres index estimates inflation of 50%, the Paasche index estimates inflation of 44.8%, and the Fischer ideal index estimates inflation to be 47.4%.

3. Consider a consumer who lives for 2 periods, who consumes in each period with CRRA period utility $u(c) = \frac{c^{\sigma-1}-1}{1-\sigma}$, discounts the future at rate β, can save (or borrow with repayment) in the first period with interest rate r, and has exogenous income in each period $w_0 > w_1$. Formulate and solve the household's lifetime utility maximization problem. If $r = 0$, how does consumption in each period change as we increase σ?

Solution:

$$\max_{\{c_0,c_1,a\}} \quad \frac{c_0^{1-\sigma}-1}{1-\sigma} + \beta\frac{c_1^{1-\sigma}-1}{1-\sigma}$$
$$\text{s.t.} \quad c_0 + a = w_0$$
$$c_1 = w_1 + (1+r)a$$

Define the Lagrangian:

$$\mathcal{L} = u(c_0) + \beta u(c_1) + \lambda_0(w_0 - c_0 - 1) + \lambda_1(w_1 + (1+r)a - c_1).$$

We get the following first order conditions:

$$[c_0] : u'(c_0) = \lambda_0$$
$$[c_1] : \beta u'(c_1) = \lambda_1$$
$$[a] : \lambda_0 = (1+r)\lambda_1$$

This implies the Euler equation: $u'(c_0) = \beta(1+r)u'(c_1)$. Using the CRRA assumption, $c_0 = [\beta(1+r)]^{\frac{-1}{\sigma}} c_1$. Let $\alpha = [\beta(1+r)]^{\frac{-1}{\sigma}}$, so the system of 3 equations which characterize the solution to the problem are

$$c_0 = \alpha c_1$$
$$c_0 + a = w_0$$
$$c_1 = w_1 + (1+r)a$$

Solving this system of 3 equations in 3 unknowns gives us the following solution for consumption:

$$c_1 = \frac{1}{1+(1+r)\alpha}(w_1 + (1+r)w_0)$$

$$c_0 = \frac{\alpha}{1+(1+r)\alpha}(w_1 + (1+r)w_0)$$

Using $r = 0$, we get that $\alpha = \beta^{-\frac{1}{\sigma}}$, and

$$c_1 = \frac{1}{1+\beta^{-\frac{1}{\sigma}}}(w_1 + w_0)$$

$$c_0 = \frac{\beta^{-\frac{1}{\sigma}}}{1+\beta^{-\frac{1}{\sigma}}}(w_1 + w_0)$$

Since $\frac{1}{1+\beta^{-\frac{1}{\sigma}}}$ is increasing in σ, we see that c_1 is increasing in σ, and thus c_0 is decreasing in σ. In the limit, $c_1 \to \frac{w_0+w_1}{2}$, as c_0 and c_1 become perfect complements.