ECON 702 Macroeconomics I

Discussion Handout 3*

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Content review

- Chapter 3 in the textbook FVK.
- Endogenous Labor Supply.
- Firm Problem.

Exercise 1. Endogenous Labor Supply

In this exercise, we consider a modified version of the household maximization problem. The new assumption is that households have a choice of how many hours to work, l_t. Now the agents will choose how much to consume, how much to save/borrow, and how much to work. The household maximization problem now reads as

$$\max_{\{c_t, a_{t+1}, l_t\}_{t=0}^{T}} \sum_{t=0}^{T} \beta^t u(c_t, l_t)$$

subject to

$$\begin{aligned}
& c_t + a_{t+1} = w_t l_t + (1 + r_t) a_t \quad \forall t = 0, 1, ..., T \\
& c_t \geq 0 \quad \forall t = 0, 1, ..., T \\
& a_{T+1} = 0 \\
& a_0 \text{ is given} \\
& l_t \geq 0 \quad \forall t = 0, 1, ..., T
\end{aligned}$$

1. The new assumption on utility function is that $u(c_t, l_t)$ is strictly decreasing in l_t $\left(\frac{\partial u(c_t, l_t)}{\partial l_t} < 0\right)$. What trade-off does this assumption create? What are other basic assumptions on u?

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2. Set up the Lagrangian and derive the intra-temporal (within-period) optimality condition. Provide an interpretation of the obtained optimality condition.

3. Find optimal labor supply using the following functional form of the utility function:

$$u(c_t, l_t) = \log\left(c_t - \psi \frac{l_t^{1+\eta}}{1+\eta}\right), \tag{1}$$

   where the parameters $\psi \geq 0$ and $\eta \geq 0$ control the disutility of work. While ψ shapes the level of disutility, η controls how fast disutility increases with l_t. How does optimal labor supply depend on wage, w_t? Does labor supply depend on the parameters ψ and η?
   Notes: the utility function (1) is known as Greenwood, Hercowitz, and Huffman (GHH) function. For this specific function, we can separate the optimal consumption (savings) choice and the optimal labor (leisure) choice.

4. Given a solution for labor, derive the optimality condition for a consumption-saving decision. Provide an interpretation for the Euler equation (intertemporal, that is between-period, optimality condition).

5. Characterize optimal consumption and assets under assumptions $(1 + r_t)\beta = 1$, $w_t = w_{t+1} = ... = w$, $r_t = r_{t+1} = ... = r$. Is the economy in a steady state?

Exercise 2. Firm Problem

1. For a neoclassical Cobb-Douglas production function

$$y_t = f(k_t, l_t) = k_t^{\alpha}(A_t l_t)^{1-\alpha}, \tag{2}$$

   where $\alpha \in (0,1)$, y_t is real output, k_t is capital, l_t is labor, and A_t denotes the level of technology (TFP), show the following properties:

   (a) the function exhibits constant returns to scale;

   (b) both inputs are essential for the production technology;

   (c) marginal product of labor and marginal product of capital are positive but decreasing;

   (d) Inada conditions are satisfied.

2. What is an interpretation of α and $1-\alpha$?

3. Consider a representative firm, owned by households, that hires workers at wage w_t per unit of time, rents capital at rate μ_t, and produces the final good. Capital wears out in production at rate δ, implying that the return on capital for households is $r_t = \mu_t - \delta$. Solve the firm profit maximization problem:

$$\max_{l_t, k_t} \{y_t - w_t l_t - \mu_t k_t\}$$

subject to

$$y_t = k_t^{\alpha}(A_t l_t)^{1-\alpha}$$
$$k_t, l_t \geq 0 \quad \forall t = 0, 1, ..., T.$$

4. Provide an interpretation of the optimality conditions for labor and capital choices.

5. Show that when a firm behaves optimally, its profits equal zero.