ECON 702 Macroeconomics I

Discussion Handout 4*

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1 Content review

- The **social planner's problem** is to maximize utility of the household subject to resource feasibility. (Assume $A_t = 1$ for notation)

$$\begin{aligned} \max_{\{c_t, a_{t+1}\}_{t=0}^T} & \sum_{t=0}^T \beta^t u(c_t) \\ \text{s.t. } & c_t + k_{t+1} = k_t^\alpha l_t^{1-\alpha} + (1-\delta)k_t, \quad \forall t \\ & c_t \geq 0, \quad \forall t \\ & 1 \geq l_t, \geq 0 \quad \forall t \\ & a_{T+1} = 0 \\ & a_0 \text{ is given} \end{aligned}$$

- The **first welfare theorem** states that if a competitive equilibrium exists and there are no externalities, market power, or imperfect information, then the competitive equilibrium allocation solves the social planner's problem.
  - This is proved by showing that the Euler equation of the household in CE matches the optimality condition of the social planner.

$$1 + r_{t+1} = \alpha k_{t+1}^{\alpha-1} + 1 - \delta$$

- The **second welfare theorem** states that the allocation which solves the social planner's problem can be implemented as a competitive equilibrium.
  - This is proved by finding some prices $\{r_t, w_t\}$ such that the allocation as a function of prices matches the solution to the social planner's problem.
- The **Golden Rule** is the steady state level of capital that maximizes period consumption, $k^g = (\frac{\alpha}{\delta})^{\frac{1}{1-\alpha}}$. However, this does not account for household discounting. The **modified Golden Rule** is the steady state capital which maximizes lifetime utility of the household including discounting, $k^* = (\frac{\alpha}{\delta+\rho})^{\frac{1}{1-\alpha}}$.

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2 Perfect complements production

Consider an environment such as that which we studied in class with a representative household (exogenous labor supply $l = 1$, initial assets $a_0 > 0$), except the final good is produced with a Leontief production function, $y_t = f(k_t, l_t) = \min\{k_t, Al_t\}$.

1. Show that the production function demonstrates constant returns to scale and that both inputs are necessary for production.
2. Define a competitive equilibrium.
3. Assume $\mu_t < 1$. What must be the capital choice of the firm in a competitive equilibrium? (Hint: the labor market must clear in equilibrium).
4. In a steady state equilibrium with $\delta = 0$, and firms earning zero profit, what are the prices in the economy? (if it helps, you can assume that $\beta \in (.5, 1)$.
5. What are the allocations for the household and the firm in a steady state competitive equilibrium?
6. Write the planner's problem.
7. Does the competitive equilibrium allocation solve the social planner's problem?