ECON 702 Macroeconomics I

Discussion Handout 6 *

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Balanced Growth Path

Suppose we have an economy in which households have CRRA utility, $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$, the production technology is Cobb-Douglas $f(A, k, \ell) = k^\alpha (A\ell)^{1-\alpha}$, with exogenous growth in total factor productivity at rate g and full depreciation $\delta = 1$. There is exogenous labor supply (normalized to $\ell = 1$), and no population growth.

1. What does it mean for a variable to be growing at a constant rate? Write A_t as a function of g, t, and A_0. What about if we were in continuous time?

2. Are the household's Euler equations, the firm's optimality conditions, or market clearing conditions any different in this environment, compared to the model we studied before? Write these conditions.

3. Define a balanced growth path.

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4. Under the conjecture that capital and consumption grow at the same rate as TFP, characterize a BGP in this economy. What is the growth rate of output?

5. Japan is a country with approximately zero population growth over the past 25 years, but their (real) GDP has grown from about 475 trillion yen to about 540 trillion yen from 1997 to 2022. What is the average annual growth rate in Japan over this time period? [1]

6. Data shows that in Japan, $\alpha \approx .43$, and real consumption in 1997 was about 258 trillion yen. [2] Using this information, give an estimate of k_{1997} and A_{1997} using our neoclassical model.

[1] From FRED data.

[2] See FRED consumption data and labor income share statistics.

Transition Dynamics

Now suppose that we do not start on a BGP.

1. Assume here that $\sigma = 1$, so $u(c) = \log c$. Let us denote our detrended variables $\hat{c}_t$, and $\hat{k}_t$ such that $c_t = (1+g)^t \hat{c}_t$ and $k_t = (1+g)^t \hat{k}_t$. Find the detrended Euler equation and resource constraint (in terms of $\hat{c}_t$ and $\hat{k}_t$).

2. Conjecture that $\hat{k}_{t+1} = s\hat{k}_t^\alpha$, and solve for s.

3. Suppose $\beta = .9$ and our other primitives are those we solved for in Japan. Solve for the policy function of capital and consumption and sketch a graph. Find the fixed point of the policy function, where the detrended variables reach a steady state.